IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Nonlinear resonant tunnelling in a novel one-dimensional semimagnetic semiconductor superlattice

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1999 J. Phys. A: Math. Gen. 32 1667

(http://iopscience.iop.org/0305-4470/32/9/013)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.118
The article was downloaded on 02/06/2010 at 08:00

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. **32** (1999) 1667–1674. Printed in the UK PII: S0305-4470(99)99095-9

Nonlinear resonant tunnelling in a novel one-dimensional semimagnetic semiconductor superlattice

Xiaoshuang Chen†, Wei Lu‡ and S C Shen‡

† Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemum-gu, Seoul 130-012, Korea

‡ National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, Chinese Academy of Science, Shanghai 200083, People's Republic of China

Received 9 November 1998

Abstract. A new two-dimensional map is proposed to investigate the nonlinear and quantum effects on the tunnelling of carriers (electrons or holes) in a novel one-dimensional semimagnetic semiconductor superlattice. The magnetic-polaron effect resulting from the exchange interaction between the carrier in a mesoscopic dot and localized magnetic-ion spins leads to a nonlinear nature of the effective Schrödinger equation for the carrier. We find gaps and different multistability in the tunnelling properties of carriers that depend critically on the wavevector of the injected carriers. In particular, when the nonlinear coefficient is increased new nontunnelling regions appear adjacent to the regular instability regions. The properties can be useful for the transmission of information in microelectronic devices.

1. Introduction

The successful molecular beam epitaxy growth of semimagnetic semiconductor heterostructures and low-dimensional quantum structures has recently stimulated an investigation of the properties of magnetic polarons formed from free carriers in low-dimensional quantum structures [1–6]. The strong exchanging interaction between the charge carriers and the magnetic ions in diluted magnetic semiconductor structures can lead to spin polarization of the magnetic ions [7], due to localized carriers, and hence to a reduction in the total energy of the system, that is, magnetic polarons are free carriers dressed by the induced magnetic polarization field of the magnetic ions. Such a complex, consisting of a charge carrier and magnetic ions with locally aligned spin, is known as a magnetic polaron. In one-dimensional systems these polaronic effects should be strong and lead to localized soliton-like states.

On the other hand, resonant tunnelling through quantum dot arrays of various geometries has been considered extensively from a physical standpoint and for its application in future nanometre electronics [8–10]. Ulloa *et al* and Wu *et al* have considered a linear array of mesoscopic potential wells separated by square potential barriers. Propagation through such an array reveals an interesting structure of tunnelling plateaus, which could form the base for a quite different type of transistor action. Experiments with such geometries have also been undertaken [11]. The phenomenon depends crucially on the linear superposition principle [10]. A natural question arises in this context, namely how spatial nonlinearities affect resonant tunnelling. Nonlinearity can arise in semimagnetic semiconductors when an electron or hole interacts with the localized magnetic-ion spins, which reacts changing the electron or hole

0305-4470/99/091667+08$19.50 © 1999 IOP Publishing Ltd

state by a feedback process [1, 2, 12, 13]. The result is often a localized excitation which extends over many atomic dimensions with an exchange field arising from the magnetic polaron interaction with a carrier state. The exchange field leads to a nonlinear term in the effective one-particle potential [1, 2, 12, 13]. The effect of a one-dimensional superlattice is to place such localized excitations in a strong periodic potential with a period comparable to the excitation characteristic length. In addition, quantum effects in low-dimensional structures are also known to be important. The nonlinear-response characteristics of low-dimensional structures are rich in properties, and unique to these periodic structures. The occurrence of nonlinearities opens up the possibility of studying new and interesting phenomena in semiconductor devices [14–16]. For example, Hawrylak *et al* have studied the tunnelling of carriers in a quantum-dot array and found the intensity-dependent gaps in the transmission spectra. In this paper, a novel one-dimensional semimagnetic semiconductor superlattice, a linear array of mesoscopic quantum dots separated by square potential barriers, is considered. We propose a new two-dimensional map method to study the effect of the nonlinearity on the resonant tunnelling of the carriers, and discuss the relation between the carrier transport properties and the injected carrier energy. In particular, we shall show that the combination of nonlinear and quantum effects leads to a very complex periodic and chaotic spatial behaviour of the carrier wavefunction. Tunnelling studies demonstrate the multistability, hysteresis and the opening of gaps in the energy spectrum.

2. Model

We consider a novel one-dimensional semimagnetic semiconductor superlattice, which is a linear array of unit cells of periodicity a, assumed with an effective radius $r_{\perp 0}$ normal to the growth direction, the z-axis. The unit cell consists of a mesoscopic dot of width d with low Mn concentration and a square barrier of width b with a high Mn concentration. Figure 1(a) shows schematically a possible realization of the proposed device geometry. The shaded areas represent the barrier region. Figure 2(b) is the potential profile of the one-dimensional semimagnetic semiconductor superlattice in growth direction. Due to the magnetically induced localization of carrier (electron or hole) in a single mesoscopic dot, in the presence of a finite initial external magnetic or nonzero internal magnetization, the exchange interaction of the carrier states of the mesoscopic dot with the localized magnetic moments of the Mn ions in the

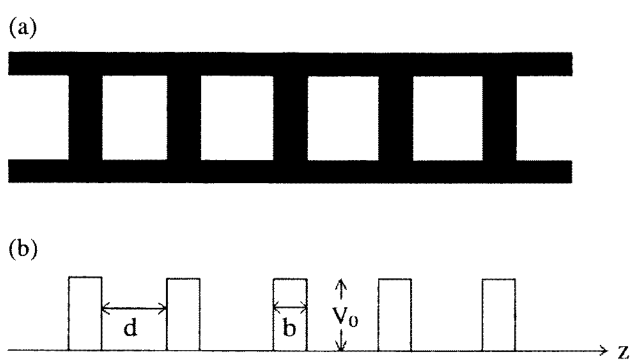

Figure 1. (a) The structural geometry of a novel one-dimensional semimagnetic semiconductor superlattice. The shaded areas represent the barrier region. (b) The potential profile of the constriction extending along the z-axis, the growth direction.

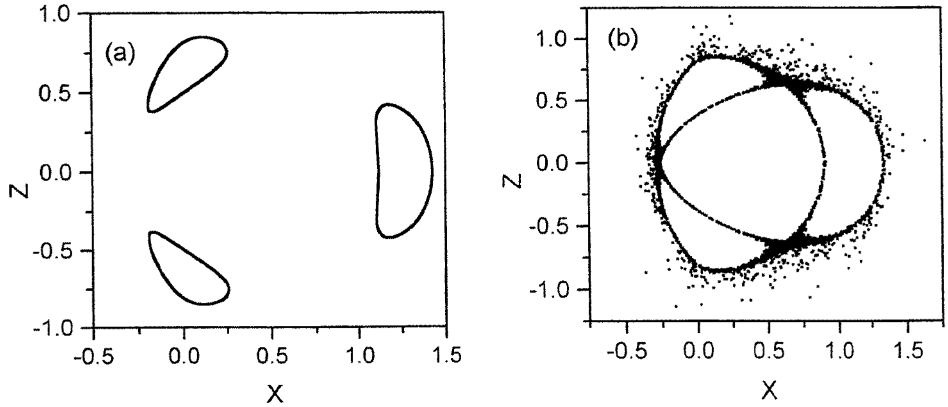

Figure 2. Orbit of the map (x_l, z_l) corresponding to the period-3 Poincaré–Brikhoff chain, with $|I_0|^2 = 1.0$, $\alpha = 10^{-2}$ Ryd*. The injected carrier wavevector ka is (*a*) 6.530 and (*b*) 6.546. In both (*a*) and (*b*) the same trajectory is plotted.

novel geometry gives rise to an effective attractive one-particle potential in the $Cd_{1-x}Mn_xTe$ barriers, which serves to further enhance the local effective fields and promotes the formation of a magnetic polaron. The exchange field arising from the magnetic polaron interaction with a carrier state will give a nonlinear term [1, 2, 12, 13]. In the one-dimensional superlattice, the antiferromagnetic interaction between Mn ions dominates in the barrier, and we expect the barrier to be in a 'spin-glass' phase. The mesoscopic dot with a low Mn concentration is assumed to be a paramagnetic phase. In the mean field approximation (MFA), an approximate Hamiltonian is derived by including the nonlinear magnetic polaronic term in the effective potential in the low Mn concentration $Cd_{1-x}Mn_xTe$ mesoscopic dot. A simple nonlinear Schrödinger equation for $\Psi(\vec{r})$ is as follows [12, 13]:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) - \left[V_0 + V_m B_{5/2}\left(\frac{g_{Mn}\mu_B B_{eff}}{kT_{eff}}\right)\right]\Psi(\vec{r}) = E\Psi(\vec{r}) \tag{1}$$

in the mesoscopic dot, and

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) = E\Psi(\vec{r}) \tag{2}$$

in the barrier. Where V_0 is the barrier height (the energy being measured from the top of the barrier), $V_m = \frac{5}{2}\sigma x\beta N_0 J_z$ is the amplitude of the exchange potential, x is the average effective concentration of Mn ions, J_z is the spin of free carrier, and σ and T_{eff} are parameters describing approximately the antiferromagnetic interaction of isolated Mn-ion spins. The Brillouin function $B_{5/2}$ describes the paramagnetic susceptibility of the Mn ion with spin $\frac{5}{2}$ in the effective magnetic field B_{eff} at the effective temperature T_{eff}. Since V_0 only depends on the variable z, we can write the wavefunction $\Psi(\vec{r}) = \phi(z)\phi(|\vec{r}_\perp|)$ with $\phi(|\vec{r}_\perp|)$ the exponential decay function of decay length $r_{\perp 0}$ and $\vec{r}_\perp$ the two-dimensional vector in the plane perpendicular to the growth z-axis. The effective field B_{eff} is a sum of the external field B_0 and the exchange field acting on the Mn-ion spin as follows:

$$B_{eff} = B_0 + B_{ex} \tag{3}$$

where the exchange field $g_{Mn}\mu_B B_{ex} = \frac{5}{2}\beta J_z|\phi(z)|^2$ arises from the magnetic polaron interaction with a carrier state $\phi(z)$. The exchange field depends on the probability $|\phi(z)|^2$ of the carrier being in the position of the magnetic ion. N_0 is the number of cations per unit volume. We note that at very low temperatures ($T < 1$ K) for the Mn concentrations of interest a spin-glass freezing takes place, which cannot be described within the MFA, and is even beyond the scope of our model Hamiltonian, which neglects long-range dipolar anisotropies. Nevertheless, such a spin-glass freezing does not qualitatively affect the static properties of the magnetic polaron, which depend essentially on local-spin correlations, and we conclude that MFA is a reasonable approximation down to $T \sim 0$ K for our problem. Actually, it is in the opposite limit of high temperature that the MFA fails to describe the magnetic polaron, because, then the magnetic fluctuations and entropy terms cannot be neglected [17, 18]. Therefore, we shall restrict our analysis to the temperature $k_B T_{eff} = 10$ meV, where the MFA is still a good approximation. Next, we shall discuss another problem, the limit of magnetic fields. On the basis of [19], we can see that the behaviour of the Brillouin function $B_J(x)$ is qualitatively similar for all values of J. It increases linearly with x for small x but saturates at 1 for large x. It represents in large magnetic fields that the Brillouin function approaches saturation and is independent of the carrier state $\phi(z)$, thus the nonlinear term in the effective Hamiltonian disappears. In this case, we recover a linear Schrödinger equation, which can be solved in the classical Kronig–Penney model. The changeover in behaviour occurs when $x \sim J$, corresponding to a field of order $1.5T$ at temperature <10 K for $J = \frac{5}{2}$. Therefore, for generality and simplicity, we take into account $k_B T_{eff} = 10$ meV, and the small or medium field, in which the linear term in the expansion of the Brillouin function $B_{5/2}(x)$ is a good approximation. In our study we limit ourselves to zero external magnetic field and narrow dots. We consider the quantum dot as an attractive potential and the carrier energy to be larger than the potential barrier. The carrier transmission in the attractive potential is well treated by Bohm [20]. Here, we mainly study the effect of a nonlinearity induced magnetic-polaron on the tunnelling of a carrier in the system of a one-dimensional superlattice. To understand the combined effects of nonlinearity and periodicity it is sufficient to replace the effective potential of the dot by a suitable chosen δ-function potential. Similar models have been proposed to study electronic transport in the nonlinear Schrödinger equations [15, 16, 21, 22]. The resulting Schrödinger equation for a system with N cells can be reduced as the following nonlinear difference equation:

$$\phi(l+1) + \phi(l-1) = 2\left[\cos(ka) - \left(\frac{V_0 a}{2} + \frac{\alpha d a}{2}|\phi(l)|^2\right)\frac{\sin ka}{ka}\right]\phi(l) \qquad (4)$$

where k is the wavevector associated with the energy $E = k^2$. In the absence of nonlinearity ($\alpha = 0$), the wavefunctions are Bloch states $= \exp(\mathrm{i}kla)$ characterized by a Bloch index k. In the nonlinear case we must retain the information about the phase of the wavefunction and the amplitude. We propose a new two-dimensional map method, in which a local transformation to polar coordinates and a subsequent grouping of pairs of adjacent variables $\phi(l-1)$, $\phi(l)$ turn equation (4) to the following two-dimensional map M:

$$\begin{aligned} x_{l+1} &= \left[2\cos(ka) - V_0 a\left(1 + \frac{1}{2}\frac{d\alpha}{V_0}(w_l + z_l)\right)\frac{\sin ka}{ka}\right](w_l + z_l) - x_l \\ z_{l+1} &= -z_l + \frac{1}{2}\frac{{x_{l+1}}^2 - x_l^2}{w_l + z_l} \end{aligned} \qquad (5)$$

where $w_l = \sqrt{(x_l^2 + z_l^2 + 4J^2}$, $x_l = 2r_l r_{l-1}\cos(\theta_l - \theta_{l-1})$, $z_l = r_l^2 - r_{l-1}^2$ with $\phi(l) = r_l \exp(\mathrm{i}\theta_l)$ and J is the conserved current, i.e., $J = r_l r_{l-1}\sin(\theta_l - \theta_{l-1})$. The map M can contain bounded and diverging orbits. The former ones correspond to transmitting waves whereas the latter correspond to waves with amplitude escaping to infinity, this means that the amplitude is diverging, and hence do not contribute to wave transmission.

3. Result and discussion

For generality we consider the injected carrier intensity to be unit. In order to analyse the carrier tunnelling properties, the orbits of the map M are studied by changing the injected carrier vector. In our calculation the dot width of 30 Å and periodic length of 50 Å are considered so that the spin-dependent Heisenberg-type interactions contain only the z components of relevant angular momenta. The barrier potential is taken from [13] with low Mn concentration ($\sim$0.1) in dot and high Mn concentration ($\sim$0.5) in barrier. The coupling constant α depends on the number of Mn ions, for example, $\alpha = 10^{-2}$ Ryd* is estimated for 10 cations per dot [16]. For zero nonlinearity, the solutions of equation (4) are plane waves, and we explicitly know the trajectories of M, linear in this case. The dynamical system included by M is integrable and the most of orbits of an integrable system are stable. The standard band structure can then be obtained from equation (4). On the other hand, for finite nonlinearity, we must retain the information about the phase and the amplitude. This problem is simplified due to the two-dimensional map M, i.e., the conservation of the current $J = r_l r_{l-1} \sin(\theta_l - \theta_{l-1})$. Thus, M becomes *a priori* a nonintegrable mapping. We find that such a mapping on the phase plane (x_l, z_l) exhibits two kinds of trajectories: bounded and diverging orbits, for example figures 2(a) and (b). The former ones correspond to transmitting carrier wavefunctions whereas the latter correspond to the wavefunctions with amplitude escaping to infinity, this means that the amplitude is diverging, and hence does not contribute to carrier transmission. In figure 2 we show one orbit corresponding to a period-(3) Poincaré–Birkhoff resonance zone, where we take the injected carrier intensity $|I_0|^2 = 1.0$, $\alpha = 10^{-2}$ Ryd* and the wavevector ka to be 6.530 and 6.546, respectively. The structure on the phase plane (x_l, z_l) is organized by a hierarchy of a periodic orbit surrounded by quasiperiodic orbits. In figure 2(a) the regular periodic orbit surrounding the three fixed points corresponds to carrier transmission through the system. In figure 2(b) on the other hand, the same trajectory is shown for a larger wavevector. We observe that a thin chaotic layer has developed that surrounds the separatrix but also that some scattered points escaping to larger z-values are visible. This trajectory corresponds to nonpassing carrier states. As the concentration of Mn ions increases, hence the coupling constant α increases, some periodic orbits become unstable and lead to stochasticity. This corresponds to passage from a carrier transmitting to a nontransmitting region. To further understand the effect of different wavevector k on the carrier transport properties, in figure 3 we show one orbit corresponding to period-(9) Poincaré–Birkhoff resonance zone with the injected carrier intensity $|I_0|^2 = 1.0$, and the wavevector ka being 6.5743 and 6.5744, respectively. We find that the number of fixed points corresponding to carrier transmission through the system is increased when the wavevector increases. The carrier transmission through the system shows different stability for different wavevectors, which is referred to here as multistability. The tunnelling studies demonstrate that different multistability appears in the one-dimensional superlattice by changing the wavevector k. We also show the same trajectory in figure 3(b) for a larger carrier wavevector and nonpassing carrier states.

In our model without exchange interaction the Schrödinger equation is linear, which can be solved in the classical Kronig–Penney model. The tunnelling can occur if the energy of the transmitted carrier is within the allowed energy spectrum of the Bloch band, irrespective of the amplitude of the carrier wavefunction or the flux of carriers. This is no longer true in the nonlinear case. In order to directly investigate the transmission properties of the injected carriers in nonlinear one-dimensional superlattice, we numerically iterate the discrete nonlinear equation (4). For the initial condition $[\phi_0, \phi_1] = [1, \exp(\mathrm{i}ka)]$ we compute the transmitted carrier wavefunction amplitude T for a one-dimensional superlattice with 10^3 nonlinear unit cells with different nonlinear parameters α and wavevectors k. In figure 4, we plot the

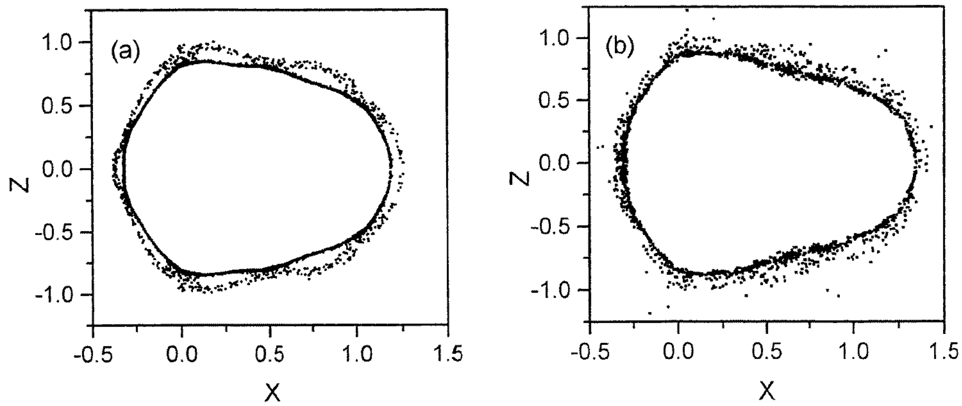

Figure 3. Orbit of the map (x_l, z_l) corresponding to the period-9 Poincaré–Brikhoff chain, with $|I_0|^2 = 1.0, \alpha = 10^{-2}$ Ryd*. The injected carrier wavevector is (*a*) 6.5743 and (*b*) 6.5744. In both (*a*) and (*b*) the same trajectory is plotted.

transmission coefficient $t = |T|^2$ as a function of input carrier wavevector k for various nonlinear values α (exchanging interaction). First, we note that there are clear transmission gaps whose width (in k space) depends on α. With α increasing the width of each gap increases and, in addition, more gaps develop in the range between two gaps. Second, this process of gap creating starts in the low-energy range and also extends, with further increased α, to the high-energy region. For example, in figure 4(*b*) the transmission gaps occur only in the low-energy range, and in figure 4(*c*) the transmission gaps also extend to a high-energy range. Furthermore, it can also be seen that more gaps appear in the transmission bands and the transmission bands become very narrow in the low-energy range. Finally, above critical α-values neighbouring gaps merge leading to a cancellation of transparency. Due to the nonlinearity, the transmitting intensity of an incident carrier on the one-dimensional superlattice is a nonlinear function of the carrier intensity. Thus the transmission coefficient as a function of the carrier intensity is not a constant. Then, for a given value of the intensity, there may be several values of the transmission coefficients. The nonlinearity leads to the multistability of the carrier transport. Also note that our effect provides a genuinely nonlinear mechanism for creating a multistable system, that is, a multilevel system.

4. Summary

We have studied the tunnelling of carriers in a novel one-dimensional semimagnetic semiconductor superlattice. The effect of the nonlinearity is considered in an effective potential in the Schrödinger equation. In general, we find that the presence of nonlinearity in the one-dimensional semimagnetic semiconductor superlattice substantially alters the tunnelling properties of the carriers. When the nonlinearity is increased new nontransmitting regions appear adjacent to the regular instability regions. Consequently, for a given carrier intensity $|I_0|^2$, an appropriate change of the wavevector of the carriers can switch the carrier tunnelling from a transmitting to a nontransmitting region. It is then possible by a simple amplitude or injected energy modulation of the injected carrier to transmit binary information to the other side of the transmission line in the forms of zeros (nontransmitting regions) and ones (transmitting region). Therefore, it is possible that these properties can be used to develop

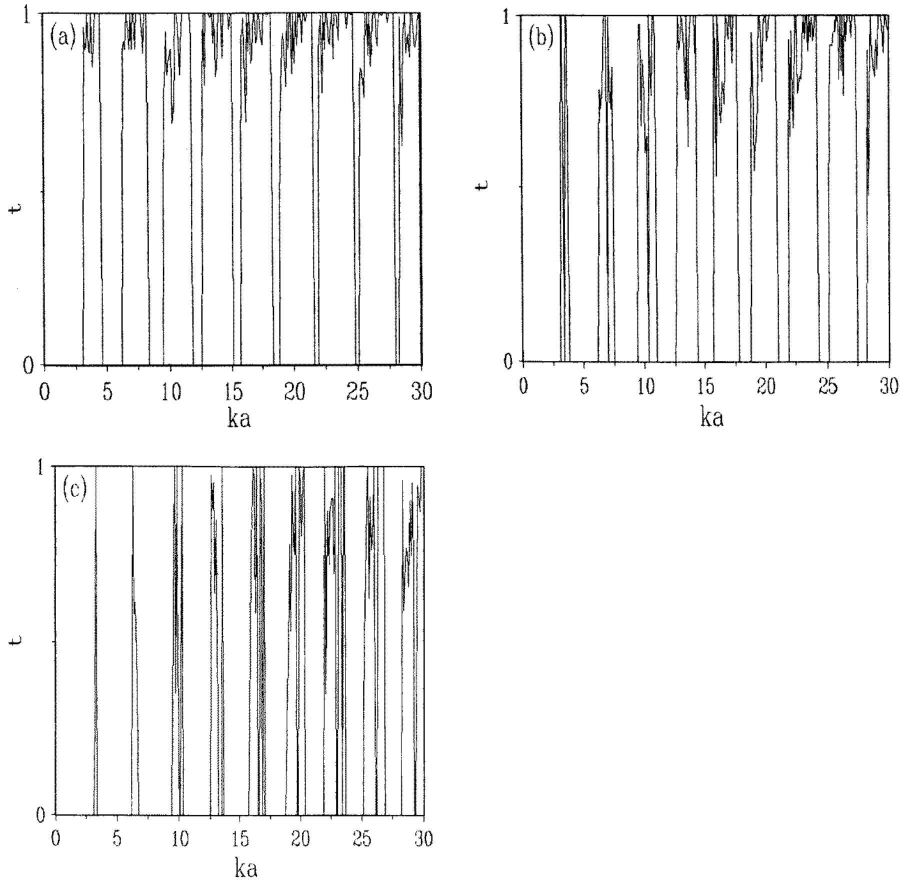

Figure 4. Transmission coefficient as a function of the wavevector k with α equal to (a) zero (linear case), (b) 0.25×10^{-2} Ryd* and (c) 10^{-2} Ryd*. The injected carrier amplitude is taken as a unit.

some new microelectronic devices for the transmission of carrier information.

Acknowledgments

This work is supported in part by the National Post-doctoral Foundation in the People's Republic of China and the QMX Project Foundation of Shanghai Science and Technology in the People's Republic of China.

References

[1] Spalek J 1984 *Phys. Rev.* B **30** 5345
[2] Zhang X C, Chang S K, Nurmikko A V, Kolodziejski L A, Gunshor R L and Datta S 1985 *Phys. Rev.* B **31** 4056
[3] Manger A, Almeida N S and Mills D L 1988 *Phys. Rev.* B **38** 1296
[4] Akinaga H, Takita K, Sasaki S, Takeyama S, Miura N, Nakayama T, Minami F and Inoue K 1992 *Phys. Rev.* B **46** 13 136
[5] Mackh G, Ossau W, Yakovlev D R, Landweher G, Hellmann R and Göbel E O 1995 *Solid State Commun.* **95** 297

[6] Yakovlev D R, Mackh G, kuhn-Heinrich B, Ossau W, Waag A, Landweher G, Hellmann R and Göbel E O 1995 *Phys. Rev.* B **52** 12 033
[7] Furdyna J K 1988 *J. Appl. Phys.* **64** R29
[8] Ulloa S E, Castano E and Kirczenow G 1990 *Phys. Rev.* B **41** 12 350
[9] Wu H, Sprung D W L, Martorell J and Klarsfeld S 1991 *Phys. Rev.* B **44** 6351
[10] Xiaoshuang Chen, Shijie Xiong and Guanghou Wang 1994 *Phys. Rev.* B **49** 14 736
[11] van Kouwenhoven L P, Hekking F W J, van Wess B J, Harmans C J P M, Timmering C E and Foxon C T 1990 *Phys. Rev. Lett.* **65** 361
[12] Heiman D, Wolff A V and Warnock J 1983 *Phys. Rev.* B **27** 4848
[13] Ji-Wei Wu, Nurmikko A V and Quinn J J 1986 *Phys. Rev.* B **34** 1080
Ji-Wei Wu, Nurmikko A V and Quinn J J 1986 *Solid State Commun.* **57** 853
[14] Delyon F, Levy Y E and Souillard B 1986 *Phys. Rev. Lett.* **57** 2010
[15] Hawrylak P, Grabowski M and Wilson P 1989 *Phys. Rev.* B **40** 6398
[16] Hawrylak P, Grabowski M and Quinn J J 1991 *Phys. Rev.* B **44** 13 082
[17] Mauger A 1983 *Phys. Rev.* B **27** 2308
[18] Dietl T and Spalek J 1983 *Phys. Rev.* B **28** 1548
[19] Callaway J 1976 *Quantum Theory of the Solid State* student edn (New York: Academic) ch 2
Hook J R and Hall H E 1991 *Solid State Physics* 2nd edn (New York: Wiley) ch 7
[20] Bohm D 1951 *Quantum Theory* (New York: Prentice-Hall)
[21] Sun N, Henning D, Molina M I and Tsironis G P 1994 *J. Phys.: Condens. Matter* **6** 7741
[22] Sun N G and Tsironis G P 1995 *Phys. Rev.* B **51** 11 221